Ruin game on random garphs

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A monopolist type ruin-game [1,2] is considered on one-component Erdos-Renyi type random networks [3]. Players with equal initial wealth are placed on the nodes of the graph. At each time-moment a monopolist game is performed on the active links. In such a game the two connected players puts a unit wealth in the pot and the pot is won with equal probability by one of them. Active links are those that connect players with non-zero wealth. The game ends when there is no active link in the graph, i.e. nodes that remain with non-zero wealth are not connected by links.

We give a compact analytical prediction for the expected number of nodes that remains with nonzero wealth at the end of the game, their wealth distribution and the duration of the game. Our theoretical prediction relies on the simple assumption that the final state of the system can be well approximated as a random selection of nodes in the graph that are not directly linked, and the distribution of wealth on these nodes is also random: each distribution of the total wealth being equally probable. The hypothesis on which our theoretical approximations are built and the theoretical predictions are confirmed by Monte Carlo type computer experiments.

We find that the number of nodes remaining with non-zero wealth depends linearly on the initial number of nodes and roughly inversely proportional with the average degree of the nodes. The wealth distribution of these nodes can be well approximated by an exponential distribution, and the expected time-duration of the game is proportional with the square of the players initial wealth and has a linear trend as a function of the logarithm of the graph size. The dependence as a function of the average degree of the nodes is more complicated, but for dense graphs it can be considered to be roughly linear. The ruin-game considered in the present study can find several applications in modeling various social or economic phenomena like: opinion formation, voting, clustering of a society according to some social and/or economic preferences or even wealth (or customer) redistribution among economic agents.

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